The Equations of Thought[1]
Jacques-Alain Miller

For Kant, what's unthinkable in the system of Spinoza can be summarized in the following proposition: "Spinozism speaks of thoughts which themselves think."[2]

Let's say that it's our acceptance and understanding of the fact that there are "thoughts which themselves think" that the discovery of Freud brings us together here.

From Fichte, the idea that there are "thoughts which themselves think" receives the name of the "postulate of unreason." Without a doubt, there's an expression that gives us pause insofar as it marks, without equivocation the limit of the philosophy of subjectivity, insofar as it is impossible for it to conceive anything of a thought that wouldn't be the act of a subject.

On the contrary, to articulate "the laws of a thought that itself thinks" requires that we constitute categories that are radically incompatible with those of the thought thought by the subject. That's why we'll be aided here by what's been elaborated in a scientific domain which has from the beginning questioned thoughts that themselves think, thoughts that are articulated in the absence of any subject to animate them.

This scientific domain is that of mathematical logic. Let's say that we will have to stick to mathematical logic as a pure logic for the theoretical game in which "the laws of a thought that itself thinks" outside of any subjectivity of the subject are reflected.

For we must note that the constitution of the domain of mathematical logic is accomplished by the progressively certain exclusion of the psychological dimension, from which it formerly seemed possible to derive the origin of the elements of specifically logical categories. Note that for us, the exclusion of the psychological leaves us free to follow, in this field, the traces that mark what we must call "the passage of the subject" according to a definition that no longer owes anything to the philosophy of the cogito insofar as it relates the concept of the subject not to its subjectivity, but to its subjection.

In what way does mathematical logic prove itself to be appropriate to our reading? Well, here's how: the autonomy and the sufficiency that it attempts to guarantee by its symbolism renders all the more manifest those articulations where the mark of its functioning stumbles. So it's quite obvious, insofar as they articulate without knowledge a suggestion of the subjectivity of the subject, that the laws of mathematical logic should engage us here.

This is what allows me to bring back, from the origins of mathematical logic, an expression whose use was abandoned a long time ago. In order to propose this expression as my subject, I am going to speak to you, partly, of the "equations of thought."

To rediscover this expression, we must push our reading beyond the formalized apparatus of modern logic. In order to rediscover it precisely with the first founder of mathematical logic—Freud was only the second—we must restore to its place the discovery of George Boole: that algebra can formulate logical relationships. This discovery is properly theoretical.

Since algebraic formalization is liberated from the field of numbers, which is no longer, then, one of its specifications, it liberates mathematical formalization, in order to state that

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[1] Jacques-Alain Miller's presentation in Jacques Lacan's seminar *The Logic of Fantasy*, session of 30 November 1966.

[2] Immanuel Kant. "What Does It Mean to Orient Oneself in Thinking?" *Religion and Rational Theology*. Ed. and trans. Allen W. Wood and George Di Giovanni. Cambridge: Cambridge UP, 1996, 1-18: 15. My translation will maintain Allen Wood's wording, but another possible reading is "thoughts that think themselves."

symbolization, properly speaking, is not dependent on the interpretation of symbols, but only on the laws of their combination.

Thus Boole attempts to establish that the laws of thought are subsumed under a mathematics in the same way as the quantitative conceptions of space and time, of number and size. However, if the first of Boole's books, *Mathematical Analysis of Logic*, is well-recognized in the field of logic as an inaugural event in its history, the second, *An Investigation of the Laws of Thought* no longer holds any place in the memory of the science of logic.[3]

Boole: In order to return to what has been abandoned in the history of logic, we will have to know what it misrecognizes in the conditions of its exercise, which will reveal to us, thereby, certain of the laws of logic that operate therein. A logic that, you will see, rises above the logician's logic. Jean-Claude Milner and I have had the occasion to present several elements of this logic, the logic of the signifier, in relation to Plato's Sophist and [Frege's] *Grundlagen*.

If I can make a presentation of it today, it is no doubt because the subject of Dr. Lacan's lectures this year have prepared the way, but also because in psychoanalysis, our formal constructions prove to be manipulable enough to be interpreted liberally within the Freudian field. One such possible interpretation eminently justifies the constitution of our symbolism and the presentation that we have made of it, as a calculus of the subject.

Moving on to Boole's doctrine, we can say right away that he doesn't innovate, since he thinks of language as the product and the instrument of thought, and he takes the sign to be arbitrary. Thus, signification is the product of a linking of a word and an idea, or rather of a word and a thing. You see right away that these two possibilities are not at all equivalent. For Boole, they are equivalent.

This means that communication is thus uniquely guaranteed by the permanence of an association, which is nothing other than the classical one and which doesn't at all go beyond the Lockean doctrine of language.

Except, we now come to the proposition that found the Boolean enterprise. All the operations of language as an instrument of reasoning could be carried out in a system of signs. But what specifies the sign as employed in algebra and logic is that it can only be a letter or a simple mark. And that's authorized by the theory of the arbitrariness of the sign. But this is the first time that we properly speaking use the sign.

Now it's necessary to learn—and that can be done quickly enough in an elementary fashion—Boole's symbolism. Let's say that there are there categories of sign to put in place:

First, symbolic letters which have the function of representing things as object of our conceptions, that mark things as objects of representation.

Second, There are signs of operation, the $+$, the $-$, the $\times$, which serve to the operations of the understanding by which our representations are combined and re-formed into new representations.

Third, and this is not the least important, the sign of identity.

1) Symbolic Letters

Let's say that the sign X or the sign Y represent a class of things to which a particular name or a property can be attributed. So we represent a circle with certain number of objects of a certain name or a certain property. We'll call this class X.

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[3] George Boole. *An Investigation of the Laws of Thought*. New York: Dover, 1958 [1854].

We'll say that the combination X × Y—we could write X • Y—represents the class of objects to which the names and the properties X and Y are simultaneously applicable: the intersection of X and Y.

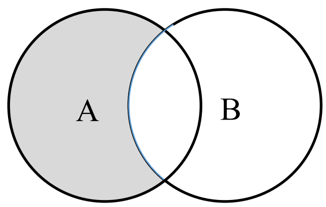
We can first of all note that the order of the symbols doesn't matter. We could write X • Y = Y • X, which is to say that symbolic letters are commutative. But Boole insists on what's involved in the laws of thought, not in the laws of nature, nor in the simple laws of arithmetic.

2) Operational Signs

Next, we get from Boole a certain number of other laws that, moreover, are not that far from the laws of arithmetic, but that represent them in logical form:

We can make use of the sign + . This will be the sign of the class that joins, for example the classes X and Y.

We can make use of the sign – , which will mark that we remove from a class a portion of its elements.



A – B

Suppose that X and Y have the same signification. As the combination of the two symbols express the set of the class of objects to with the names or the properties represented by X and Y are jointly applicable, this combination expresses nothing more than one of the two symbols.

This appears very simple. You're going to see with what ingenuity Boole draws from this a law that he calls fundamental for thought.

If the two symbols say nothing more than one of the two, then X • Y = X, and since Y has the same signification as X, we can state that X • X = X.

This is particularly simple. We can even write it by applying a rule that will translate the symbolism. We can write this law in all simplicity, $X^2 = X$.

Since all of this is extraordinarily simple, we have to try—each time—to emphasize how important it is. This formula, $X^2 = X$, is given in the algebra of logic as a major law of thought. What we must say about it is that it governs in some way everything that we can define as belonging to the dimension of signification.

First we have to remember that in algebraic logic all the symbols that are to take on value are subjected to this law as a representation of the laws of thought. Even if there isn't a subject common to the laws of logic and arithmetic, there is a community of formal laws. And that's what the Boolean algebra is founded upon.

That is why we have to find, as soon as we have this formula, an interpretation through numbers. For it is immediately apparent that arithmetically only two numbers are capable of interpreting this formula in a satisfactory way. It's quite obvious that the only two numbers that can interpret this formula are 0 and 1.

Still, you mustn't get the idea that every X that we will encounter in logic, in the logic of thought, has to be interpreted by 0 or 1. But we do have to say that only 0 and 1 respond, in numeration, to the Boolean law of thought that we've called the law of signification. From here on out, let's say that it's arithmetic that going to guide logic.

Let's examine the arithmetic properties of 0.

Put simply, $0 \cdot Y = 0$, whatever it is that Y represents.

This means that the class 0 multiplied by Y is identical to the class represented by 0.

In other words, there is only one possible interpretation of 0: 0 represents nothing, but this 0 that represents nothing is a class.

Now let's examine the arithmetic properties of 1: $1 \cdot Y = Y$.

The symbol 1 represents and only represents a class such that all the individuals, no matter what the class X is, are also its members. The result: this class can only be the universe, defined as the class in which is included all the individuals of no matter what class.

What you see appearing here is the category of the universe of discourse, about which Dr. Lacan talked to you last time.

You see it here deduced in Boole through the most elementary symbolism.

Let's follow the elaboration of Boole.

Now, take X. It doesn't matter what class. If 1 represents the universe, it is clear that $1 - X$ is the complement of X: it's the class made up of objects that are not included in X. We're going to perform a very simple transformation on the formula $X^2 = X$. It suffices to make one of the members of this equation pass to the other side of the = sign.

You're going to have two possibilities. Boole chooses only one of them. We can obviously put X on the side of X^2 or do the opposite. Boole chooses only one of these two possibilities and the other disappears. It will never be spoken of again.

$$X - X^2 = 0$$

This is the derivation of the transformation that Boole chooses. And he deduces from it another formula just as simple:

$$X \cdot (1 - X) = 0$$

There is no intersection between $1 - X$ and X, which is to say, quite simply, that it is impossible for a being to possess a quality and to not possess it at the same time.

Starting from the las $X^2 = X$, we derive from it by this interpretation and enunciation of the principle of contradiction, given by Boole as a consequence of the fundamental equation of thought.

In other words, in the order that he follows, the constitution of thought is prior to the principle of contradiction.

We can say that these X's and these Y's have been interpreted as being in classes, but could have been interpreted otherwise.

Under these conditions, what is the multiplication which we perform on X^2, the multiplication of X by itself, but the operation by which one thing, any thing, comes to be signified to itself, and by which any sign comes to signify itself?

3. The Sign of Identity

The formula $X^2 = X$ is a more elaborated form than the formulation of the principle of identity. But it's a formula that makes something emerge to which we shouldn't be indifferent: identity supposes the duality of the identical element in the operation of signifying itself. This means that there is no self-identity without alterity. For those who are familiar with Dr. Lacan's system, such a proposition has resonance.

In other words, what interest can we take in Boole's equation? It's this: that it reveals by its formula, $X^2 = X$, that the signification of an element in a *universe of discourse*, implies its reduplication and that it's self-identity is nothing but the reduction of its double to itself.

In order to situate these ideas, let's say—following Boole—that this "law of signification," the "fundamental law of thought," as Boole says, is an equation of the second degree. That's obviously the most concise formulation that one could give of a principle that has in some way governed a good part of Western philosophy.

The fact that in signification, thought doesn't work without following this second-degree equation, means that the dichotomy is the process of all analysis of signification, from which we can deduce—we won't do it here, but it's simple enough—that binarism isn't a contemporary manifestation of reflection, of analysis, but rather, it's already inscribed in this duality.

Boole refuses to make a supposition, by saying that one can't conceive of a thought that would be governed or expressed by a third degree equation. One can't even conceive of what one would be. Why isn't the equation $X = X^3$, for example, interpretable in algebraic logic? It isn't interpretable because in whatever way we transform this equation, it calls into question two terms that aren't interpretable in algebraic logic, on the one hand, the expression (and we must take note of the word "expression") $1 + X$, and on the other hand the symbol -1.

For we can already make the symbol -1 appear prematurely in the derivation that Boole didn't perform with his formula. In effect, what he chose to say was $X - X^2 = 0$. If he had said, $X^2 - X = 0$, we'd have $X \cdot (X - 1) = 0$, and the -1 would already have been present.

He excluded one of the two possible transformations!

It's only at the level of $X = X^3$ that he rediscovers the -1. Why must the symbol—I don't mean here the interpretation that one gives it of "universe"—why must the symbol itself, -1, be excluded from the field of logic?

Quite simply because it doesn't follow the law $X^2 = X$. In other words, in order to draw the simplest most immediate conclusion from Boole's text, at the origin of mathematical logic, at the very point where it is founded, the exclusion of -1 is accomplished.

Why? According to the law, because it's the very symbol of non-self-identical, insofar as it doesn't follow the law of identity, of non-contradiction in the order of signification.

Why is the expression $1 + X$ excluded? Boole says it's excluded because one can't conceive of the addition of anything to the universe. For, in $1 + X$, the 1 represents the universe, and X is the element that appears in excess [*en surcroit*] of the universe. In fact, in the formula, $1 + X$, it's the X that represents a unit, a unique element.

Thus what can't be accepted in mathematical logic, to the point that it is truly constituted, is the excess [*l'excès*]of an element in the universe, the excess of what one could call a $+1$, or a one-more [*1 en plus*].

So let's say, simply enough, that we've spoken prematurely of the -1, that at the origin of mathematical logic, the exclusion of $+1$ is accomplished, the symbol of outside of signification [*hors signification*] or outside the signified [*hors signifié*] and of the non-representable insofar as it exceeds the totality of the universe.

It could be shown that these two exclusions are really one: that the excess 1 and the lack of 1 [*le "1 par excès" et le "1 par défaut"*] occupy the same place both in relation to signification and to reality. That is, both in relation to the universe of discourse and to the universe of things that responds to it.

We could express the conjunction of these two exclusions, their unity, in the following formula: In the order of signification, the extra is lacking [*l'en-plus manqué*].

Without going much further, we could formulate here, let's call it, a law of the sign, as an element of signification. It suffices to say that in signification, the signs endowed with signification are constituted in such a way as to obey Boole's law, but that the signifier, as the material of the sign, or as an element outside the signified, doesn't obey it.

Here we finally rediscover the oft-repeated axiom, "the signifier doesn't signify itself," which is properly speaking the opposite of Boole's law, but that allows us to understand that the signifier is not constituted in the image of the signification that it supports.

Here we can give a simple formulation, in order to remind ourselves, that the multiplication of -1 by itself doesn't give us back -1.

But if we want that inverse multiplication (Boole interpreted it this way: -1(-1) = 1 + 1′′), the factor, let's interpret it thus, institutes the order of the signified as the inverse of the order of the signifier insofar as the signifier repeats itself and can only repeat itself (-1, -1, -1, . . .) whereas the signification can be multiplied, that is, redoubled.

Let's flesh out what perhaps hasn't any image—that the chain of signifiers must be thought as constituted by a concatenation of -1, by units constituted as -1, as "catenations" but let's say that they are *units* so as to generalize the reference of Dr. Lacan to *units of the unary type*.

We've produced or made appear the category of + or -1. Now we must understand exactly by that path is imposed the order of signification. In order to take up these two laws, that of the sign and that of the signifier, we have to show that + *or* -1 is produced by any signification insofar as it posits and operation of redoubling.

In order to state it, we could start from the relations of thought to consciousness and, let's say, from what's called *reflection*. To understand that, we could first go search for a mathematical definition of reflection or reflexivity. Let's borrow it from Russell's *Introduction to Mathematical Philosophy*.[4]

What he says is simple: A class—perhaps we should say a collection or a set—is reflexive if it is a class similar to a part of itself, which means that part of this collection could mirror the whole, or rather, that the similarity between the two sets, the part and the whole, consists in the possibility of linking to any element of the whole an element of the part, or of placing them in a bi-univocal correspondence. Reflexivity is a property of an infinite set. We can exemplify it by the countable infinity of the "whole," the natural numbers.

We can link to any natural number an even number. That is to say, we could make 1 correspond to 2, 2 to 4, 3 to 6, and so on to infinity. We can apply the set of all the even and odd numbers to just the even numbers. There are, if you like, the same number of even numbers, on the one hand, as there are odds, on the other. That's a property that characterizes the infinite set.

Let's say that what characterizes the cardinal number of this collection, to give it is simple characteristic, is that it remains unchanged by the addition or subtraction of one unit or several.

Take one unit. What characterizes, say, the number n of such a set is that $n = n + 1$ as well as $n = n - 1$. What's more, the two propositions mean exactly the same thing. That is elementary in the theory. I mention it only to mark and to punctuate +1 and -1.

If there is in Spinoza thoughts that think themselves in the divine understanding, it's precisely because the divine understanding is infinite. Such that there are as many ideas as there are ideas of ideas, and so on. In the same way as even numbers are ideas of ideas, the numbers even and odd are the sum of the ideas and the ideas that reflect them.

If God has a consciousness of his ideas, but not a consciousness of himself, that means that he is not a person. He has consciousness of his ideas by the property of reflection of the

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[4] Bertrand Russell. *Introduction to Mathematical Philosophy*. New York: Dover, 1993 [1920].

infinite set of his infinite understanding. However, if there is something that we call an "whole" and something that we call a part, there must be at least a small difference between one and the other, the simple difference that maintains the difference between part and whole.

This set must respond to the law: $n = n - 1$. Let's say, to be more clear, that there is only a reflection if something of the "whole" falls outside of this reflection, an element of the whole. We see this when we place all the natural numbers in correspondence with all the natural numbers -1.

It's necessary to skip at least one element at the start in order for there to be this inflection, for it to have a meaning. Not to mention that often it's the 0 of the series that corresponds to the 1. Thus, the 0 has no reflection. Suffice it to say that an element falls. And what does it represent, this element that falls? It represents the difference between the whole and the part. That means that in some way the whole itself falls, or the totality of the whole.

In other words, to have consciousness of his ideas of the Spinozian type implies that there is no consciousness and that there is an infinite understanding. Of course, that relies on the type of reflection that Sartre calls, the necessity of reflection as positional consciousness. That supposes the model of a bi-univocal link to an idea, and to the consciousness of an idea. That supposes a bi-univocal link between the idea and the idea of the idea, on the model of Spinozian reflection.

For, in *Being and Nothingness* (pp. 3-17),[5] Sartre demands that we avoid what he calls an "infinite regress." He has no other word with which to condemn such an infinite regress than "absurd": "If we wish to avoid an infinite regress, there must be an immediate, non-cognitive relation of the self to itself" (12). We could formulate this proposition in terms that are not quite those of Sartre's to advance it even more clearly.

Sartre says, "If we wish to avoid. . . ." If we exclude the possibility of an infinite understanding and if we want to obtain self-consciousness, we have to produce a reflection, an element such that it is related to the self without being reduplicated. This, said Sartre, is the non-thetic, non-positional self-consciousness of the type opposed to the Spinozian, which no longer supposes an element here and an element there.

He writes, "We understand now why the first consciousness of consciousness is not positional; it is because it is one with the consciousness of which it is consciousness" (13-14).

By taking this text with brutal literalness, at face value, and by imposing on Sartre a schema that is not his own, that of univocality, if we try to think the text of Sartre starting from the linkage of bi-univocality in reflection, we have to say that if the element called "consciousness of consciousness" is one with the consciousness of which it is conscious, if there truly is a possibility of a unity of the one and the other, this element called consciousness of consciousness, or non-positional self-consciousness is constituted as an ego [*moi*], an ego that, Sartre says, takes its disguises from its want-to-be [*manqué à être*], another formula that I haven't noted here.

At the same time, if something like "a consciousness of consciousness" manifests itself, we must say that in the field of reflection it is an aberrant phenomenon, an odd or extra element coming to break the bi-univocal correspondence of ideas and of the ideas of the idea.

What is there to say about this element "consciousness of consciousness" if not that it takes the position of a point of reflection such that it supports the difference between the whole and the part all by itself. All by itself it takes on the reflective property of the infinite collection.

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[5] Jean-Paul Sartre. *Being and Nothingness*. Trans. Hazel E. Barnes. New York: Washington Square Press, 1956 [1943].

This point is in some way, in conscious thought, in its space, an infinite point. That's what comes to crush the infinite collection posited by Spinoza.

The aberrations and the lack of this point are somewhat similar to a category that Sartre uses here and there regarding bad faith, which is the category of evanescence. This point is evanescent. . . . Or rather say that this point in reflection necessarily *vacillates* between +1 and -1.

In this vacillation, we have to recognize an obviously heterogeneous being, in reflection as well as in reality a being always *in excess* [*de surcroit*] of reality and the reflection with which it comes to be identified, and always *short of* [*en défaut*]reality when separated from it. Let's call this heterogeneous being the being of the subject.

It was our intention to complete this somewhat by examining the principle of a vicious circle, or we may refer, let's say, to naked being, the birth of this +1, produced by the one-more [*1 en plus*] produced by signification. To go quickly, let's say that the principle, and everything that supports the set or a collection must not be an element of the collection. What forms a set from a collection can't be interior to the collection.

Which means we can only predicate a collection on its exterior, or again, we can't think the unity of a collection except from outside of it.

To take a collection to be a set assumes that it is enclosed. This enclosure is itself the unity of the collection. The enclosure of any collection is an element produced as well by any predication, any discourse on the collection. The collection can only be signified as such starting from the one-more.

Starting from this formula, we could also arrive at this: The one-more must be lacking in the elements of a collection for it to be closed.

We could interpret that as an uncountable element, outside of signification, to which the signification returns, insofar as it superimposes a redoubling. This shows how we have to contradict Boole's equation, even though it remains fundamental.

We could complete it by a test of Russell's theory of types. But this test has already been carried out in part by Dr. Lacan on the "I am lying" that he sees produced, according to Russell's theory of types, by the division of the subject. The "I am lying" could be included in the truth, in the element of truth, on the condition of redoubling the "I."

This division of the subject is produced by truth, this division of the subject responds in way that's a bit inflected by Bachelard's formula: "Every value divides the valorizing subject."

I believe I've said enough about this division of the subject that it shouldn't be confused—this is important to the theory—with the reduplication of signification.

—translated by Dan Collins